Simulation and Model Validation of a Parabolic Trough Solar Collector for Water Heating

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Abstract: The aim of this study is to analyze the performance of a parabolic trough solar collector (PTC) for water heating and to validate the model performance. The simulated model was compared, calibrated and verified with the experimental results. RMSE (Root mean square error) was used to calibrate the convective heat transfer coefficient between the absorber pipe and the ambient air which was the main factor affecting the heat transfer associated with the PTC. The calibrated model was better fitted with the experimental model. The maximum, minimum and mean deviation between the measured and predicted water temperatures differed only 0.81°C, 0.09°C and 0.31°C respectively in the calibrated model. RMSE values were decreased from 0.5389 to 0.4910, 0.0134 to 0.0125 and R-squared was increased from 0.9955 to 0.9956 after calibration. The temperature of water was increased from 33.7°C to 48°C in 12 hour test. The thermal efficiency of the collector was calculated to be 55%. The calibrated model showed good agreement with the experimental data for model validation.

Key Words: Parabolic trough collector, Solar water heating, Simulation, Model validation.

Nomenclature

$A_a$: area of the absorber pipe [m$^2$]
$c_w$: specific heat of the working fluid [$Jkg^{-1}C^{-1}$]
$c_{pw}$: specific heat of the water in the storage tank [$Jkg^{-1}C^{-1}$]
$A_s$: area of the storage tank exposed to the ambient air, 8 [m$^2$]
$A_w$: area of the working fluid [m$^2$]
$d_{si}$: inside diameter of the absorber pipe [m]
$CR$: concentration ratio
$C$: concentration
$e_a$: specific heat of the absorber pipe [$Jkg^{-1}C^{-1}$]
$d_{so}$: outside diameter of the absorber pipe [m]

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\[ d_i \] : difference between ith estimated and ith measured values

\[ h_i \] : convective heat transfer coefficient between the working fluid and the absorber pipe \([\text{Wm}^2\text{C}^{-1}]\)

\[ h_{r,sky} \] : radiative coefficient between the absorber pipe and the surroundings \([\text{Wm}^2\text{C}^{-1}]\)

\[ h_w \] : convective heat transfer coefficient between the absorber pipe and the air \([\text{Wm}^2\text{C}^{-1}]\)

\[ I \] : solar intensity \([\text{Wm}^2]\)

\[ k_a \] : thermal conductivity of the absorber pipe \([\text{Wm}^{-1}\text{K}^{-1}]\)

\[ L \] : length of the absorber pipe \([\text{m}]\)

\[ m_s \] : mass of the water in the storage tank, 200 \([\text{kg}]\)

\[ m_w \] : flow rate of the working fluid \([\text{kgs}^{-1}]\)

\[ n \] : number of data pairs

\[ \text{Nu} \] : Nusselt number

\[ Pr \] : Prandtl number

\[ Re \] : Reynolds number

\[ T_a \] : absorber pipe temperature \([\text{C}]\)

\[ T_e \] : air temperature \([\text{C}]\)

\[ T_s \] : water temperature in the storage tank \([\text{C}]\)

\[ T_{sky} \] : effective sky (lower atmosphere) temperature \([\text{C}]\)

\[ T_w \] : working fluid temperature \([\text{C}]\)

\[ t \] : time \([\text{seconds}]\)

\[ U_L \] : heat loss coefficient of PTC \([\text{Wm}^2\text{C}^{-1}]\)

\[ U_s \] : overall heat transfer coefficient from the storage tank to the ambient air \([\text{Wm}^2\text{C}^{-1}]\)

\[ V \] : wind speed \([\text{m/s}]\)

\[ w \] : reflector aperture \([\text{m}]\)

\[ X_i \] : ith measured value

\[ x \] : distance \([\text{m}]\)

\[ Y_i \] : ith measured value

\[ \gamma_r \] : radiative reflection factor

\[ \epsilon_a \] : emissivity of the absorber pipe, 0.9

\[ \eta_c \] : thermal efficiency of the PTC

\[ \eta_o \] : optical efficiency, 0.65

\[ \mu_w \] : viscosity of the working fluid \([\text{kgm}^{-1}\text{s}^{-1}]\)

\[ \rho_a \] : density of the absorber pipe \([\text{kgm}^{-3}]\)

\[ \rho_w \] : density of the working fluid \([\text{kgm}^{-3}]\)

\[ \sigma \] : Stefan-Boltzmann constant, \(5.6697 \times 10^{-8} \ [\text{Wm}^{-2}\text{K}^{-4}]\)

\[ \tau_a \] : absorptance-transmittance product, 0.9

1. Introduction

Burning fuel and coal for the heating of buildings has caused significant environmental pollution and greenhouse gaseous emissions. Solar water heating technology is eco-friendly and can be used for reduction of greenhouse gases and environment pollution [1]. Solar water heating technology has been frequently applied for building heating and cooling purposes, domestic water heating, sea water desalination, drying of biomaterials, and
industrial purposes [2].

Solar collector is usually used for water heating. It transfers the solar energy into thermal energy. Parabolic trough solar collector (PTC) technology is the most viable and advanced of the solar thermal technologies [3, 4]. It can be used effectively and efficiently for industrial purposes. PTC was employed frequently for solar steam production [5-7], for power generation [8, 9], or for desalination [10-12] because it is cheaper and process heat application up to 400 °C could be easily obtained [4]. Mathematical analysis of the PTC was performed [13-16] and efficiency evaluation was conducted as well [6, 8, 17, 18]. Mathematical equation can be simulated for the performance, design and optimization of thermal processes [19]. The simulation can be successfully used for long term performance prediction of PTC operation [20].

PTC has been successfully employed for water heating system and simulation is being used for its model validation. Research conducted by [21] has shown that temperature of water in storage tank could be increased from 35 °C to 73.84 °C within 16 hrs at the average beam radiation of 699 W/m². The experimental testing of locally fabricated PTC with simple tracking mechanisms in Arabia[22] has shown 40% collection efficiency. Similarly, experiment performed by [23], for performance analysis of PTC water heating system had shown 55% collector efficiency. Kalogirou [20] concluded that there was only 7% variation in experimental and simulated results. Therefore, simulation can be effective method for model validation of PTC. In this research, the simulated result was compared, calibrated and verified with the experimental data and calibration of the convective heat transfer coefficient between the absorber pipe and the ambient air which was the main factor affecting the heat transfer associated with the PTC was performed for better model fit.

2. Mathematical Models

The system for simulating PTC is given in Fig.1. The system boundary of the PTC is the absorber pipe, working fluid and the storage tank. The collector consists of the cylindrical parabolic reflector and the absorber pipe. The absorber pipe is centered along the reflector’s focal line. The temperatures of the absorber pipe are assumed to be uniform. The temperature gradients through the thickness of the walls of the absorber
are assumed to be negligible. The working fluid is assumed to be completely filled in the liquid phase. The mass flow rate of the working fluid is considered to be constant and the heat transfer from the absorber pipe to the working fluid is of convective nature. Under the assumptions stated above and with the finite control volumes around the absorber pipe, the energy balances for the absorber pipe, the working fluid and the storage tank can be written as follows:

Absorber pipe [15]

\[ A_j h_a \frac{\partial^2 T_a}{\partial x^2} + \pi \gamma_c Jw - \pi d_a (h_w (T_a - T_w)) + h_{r, slab} (T_a - T_{slab}) - \pi d_a h_i (T_a - T_w) = \rho c_w A_w \frac{\partial T_w}{\partial t} \]  

(1)

\[ h_w = 2.8 + 3.0 V \]  

(1.1)

\[ h_{r, slab} = c_\sigma (T_a^2 + T_{slab}^2) (T_a + T_{slab}) \]  

(1.2)

\[ h_i = \frac{Nw \cdot k_w}{L} \]  

(1.3)

\[ N_u = 0.023 Re^{0.8} Pr^{0.4} \]  

(1.4)

\[ Re = \frac{4 m_w}{\pi d_a \mu_w} \]  

(1.4)

Working fluid

\[ \rho_w c_w A_w \frac{\partial T_w}{\partial t} + m_w c_w \frac{\partial T_w}{\partial x} = \pi d_a h_i (T_a - T_w) \]  

(2)

Storage tank

\[ m_c \frac{dT_s}{dt} = m_w c_w (T_w - T_s) - U_s A_s (T_s - T_a) \]  

(3)

Thermal efficiency of PTC

PTC is inherently more efficient at a given temperature than flat plate collector since the area from which heat is lost is smaller than the aperture area. Thermal efficiency of the PTC is defined as:

\[ \eta_e = \frac{U_i (T_s - T_a)}{I \times CR} \]  

(4)

3. Experimental Methods

The lab scale PTC was fabricated and installed on the roof of three story building located in Chuncheon, Korea. System consists of parabolic trough solar collector, the absorber pipe, the working fluid and the storage tank. System specifications are shown in Table 1. Ethylene glycol was used as a working fluid. Working fluid was circulated from the storage tank passing through the absorber pipe and then back to the storage tank by a pump. The solar energy absorbed by the absorber pipe was transmitted to the working fluid through convection. Then, heated fluid was passed through a heat exchanger transferring heat to the water in the storage tank. Accordingly, cycle continued and water in the storage
tank was heated. Sun tacking system was installed as well to obtain the maximum solar energy. And the concentration ratio was calculated as follows:

\[
CR = \frac{\text{parabola aperture}}{\text{outer diameter of absorber}}
\]

Experimental results were compared with predicted values under the same meteorological conditions and PTC characteristics. The predicted and measured temperature of the water in the storage tank is shown in Fig. 2. The temperature of water in the storage tank was increased from 32.7 ºC to 48 ºC in 12 hour test. The average solar radiation was 539 Wm\(^{-2}\) which peaked up to 934 Wm\(^{-2}\) between 12 and 1 PM. The thermal efficiency of the collector was 55%. Both the predicted and measured temperatures followed the same trend (Fig.2). The temperature deviations in maximum, minimum and mean values were 1 ºC, 0.1 ºC and 0.39 ºC respectively as shown in Table 2.

The temperature of water in the storage tank was measured periodically by the thermocouples mounted inside of the storage tank. The initial temperature of water in the tank was 33.7 ºC. Experiment was performed on 09/14/2010 and the data collected were used for the simulation of the model. Matlab v. 7.0.4 (Mathworks, Inc) was used to solve the energy balance model equations. The solar radiation, wind speed, ambient air temperature, and relative humidity were recorded in the Weather Stations (Watch Dog 2000 Series, spectrum Technologies, Inc. Plainfield, IL, USA). All the parameters were measured as a function of time.

### 4. Results and Discussion

Experimental results were compared with predicted values under the same meteorological conditions and PTC characteristics. The predicted and measured temperature of the water in the storage tank is shown in Fig. 2. The temperature of water in the storage tank was increased from 32.7 ºC to 48 ºC in 12 hour test. The average solar radiation was 539 Wm\(^{-2}\) which peaked up to 934 Wm\(^{-2}\) between 12 and 1 PM. The thermal efficiency of the collector was 55%. Both the predicted and measured temperatures followed the same trend (Fig.2). The temperature deviations in maximum, minimum and mean values were 1 ºC, 0.1 ºC and 0.39 ºC respectively as shown in Table 2.
Table 2. Derivation of measured and predicted temperature with the uncalibrated model

<table>
<thead>
<tr>
<th></th>
<th>Predicted (°C)</th>
<th>Measured (°C)</th>
<th>Deviation (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>33.4</td>
<td>33.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum</td>
<td>48</td>
<td>49</td>
<td>1</td>
</tr>
<tr>
<td>Mean</td>
<td>40.89</td>
<td>41.28</td>
<td>0.39</td>
</tr>
<tr>
<td>Range</td>
<td>14.85</td>
<td>15.5</td>
<td>0.65</td>
</tr>
</tbody>
</table>

4.1 Calibration of model

Different statistical indicators have been used to evaluate models. Statistical parameters such as root mean square error (RMSE) and R-squared for the model evaluation were discussed in [24, 26].

\[ RMSE_1 = \left( \frac{1}{n} \sum_{i=1}^{n} e_i^2 \right)^{\frac{1}{2}} \]  

The results are better with lower values of RMSE1. In case of cumulative errors, RMSE1 could be increased which is the one of the major demerits. To cope with problems, [24, 26] employed relative root mean square normalized deviation.

\[ RMSE_2 = \left( \frac{1}{n} \sum_{i=1}^{n} \left( \frac{d_i}{Y_i} \right)^2 \right)^{\frac{1}{2}} \]  

Accordingly, the third indicators R-squared is a statistical measure of how well a predicted line approximates measured data.

\[ R-squared = \left( \frac{n \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \sqrt{n \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2}} \right)^2 \]  

R-squared value equal to 1 implies that model provides perfect prediction and 0 implies that there is no relationship between measured and predicted values.

Radiative reflection factor (or radiative interception factor) of the absorber pipe which is the ratio of the radiative reflectivity to the absorbed solar radiation from the material specification was 0.9±2%. The actual value of radiative reflection factor was taken as 0.9 on the basis of lower RMSE value as shown in Fig. 3.

![Fig. 3 Sensitivity of RMSE to radiative reflection factor.](image)

Table 3. Calibrated model coefficient

<table>
<thead>
<tr>
<th>Model calibration</th>
<th>original</th>
<th>calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>convective heat transfer coefficient between the absorber pipe and the ambient air (k_a)</td>
<td>(2.8 + 3V)</td>
<td>(2.2 + 2.4V)</td>
</tr>
</tbody>
</table>
The uncalibrated model predictions depend upon the assumed value for convective heat transfer coefficient between the absorber pipe and the air. Mathematical terms for long wave radiation and convective heat transfer between the working fluid and the absorber pipe do not depend on the weather conditions, especially wind speed, so that they are given constant based on the time. Therefore, calibration was performed using the convective heat transfer coefficients between the absorber pipe and the air by minimizing RMSE values. The final calibrated value is given in Table 3 and comparison of predicted and measured water temperature in the storage tank from the calibrated model is represented in Fig. 4.

After the calibration, the maximum, minimum and mean temperature deviation between the measured and predicted temperature reduced from 1 °C to 0.81 °C, 0.1 °C to 0.09 °C and 0.39 °C to 0.31 °C respectively as shown in Table 4. Also, there was no apparent bias between the measured and predicted temperature as shown in Fig. 5. In the calibrated model, RMSE1, RMSE2, and R-squared value was improved from 0.5389 to 0.4910, 0.0134 to 0.0125 and 0.9955 to 0.9956 respectively represented in Table 5.

![Fig. 4 Comparison of predicted and measured water temperature in the storage tank from the calibrated model.](image)

![Fig. 5 predicted and measured water temperature from the uncalibrated and calibrated model (Lines are 1:1).](image)

<table>
<thead>
<tr>
<th>Statistical indicators</th>
<th>Original</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE1</td>
<td>0.5388</td>
<td>0.4910</td>
</tr>
<tr>
<td>RMSE2</td>
<td>0.0134</td>
<td>0.0125</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.9955</td>
<td>0.9956</td>
</tr>
</tbody>
</table>

**Table 5.** Derivation of measured and predicted temperature after calibration

<table>
<thead>
<tr>
<th></th>
<th>Measured (°C)</th>
<th>Predicted (°C)</th>
<th>Deviation (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>33.41</td>
<td>33.50</td>
<td>0.09</td>
</tr>
<tr>
<td>Maximum</td>
<td>48.19</td>
<td>49</td>
<td>0.81</td>
</tr>
<tr>
<td>Mean</td>
<td>40.97</td>
<td>41.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Range</td>
<td>14.78</td>
<td>15.50</td>
<td>0.72</td>
</tr>
</tbody>
</table>
4.2 Validation of calibrated model

The calibrated model was tested using the separate data set collected on 10/07/2010 using the same PTC. Fig. 6 shows the validation data set used in the calibrated model.

The statistical indicators RMSE1, RMSE2 values were reduced from 0.4732 to 0.4333 and 0.13 to 0.012 respectively and R-squared value was increased from 0.9817 to 0.9818 after calibration. The statistical errors are shown in Table 6. Also, there was no apparent bias between the measured and predicted temperature as shown in Fig. 7.

5. Conclusions

In the present work, the energy balance equation of PTC for water heating was simulated and compared with experimental results. The simulated and experimental results were in good agreements. The published value of heat transfer coefficient between the absorber pipe and the air was calibrated by minimizing the root mean square error (RMSE) from the predicted and measured temperature. Calibrated model was verified using the independent experimental data set. The minimum and the maximum deviations between measured and simulated values were reduced from 0.29% to 0.26% and 2.04% to 1.65% respectively after calibration. The results showed that the developed calibrated model could predict the PTC system performance with higher accuracy. Thus, developed calibrated model can be simulated and employed for the long term performance prediction of parabolic trough collector for water heating system.
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References


